Small Signal Gain Calculations for High-Flow CO Discharge Lasers

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Theme

A SELF-CONSISTENT theoretical model has been developed for high flow CO-He-N_2 laser systems. This model couples the kinetics of the electrons and heavy particles with the fluid dynamic processes in the laser system. Moreover, for each E/N (where E is the electric field and N is the number density) and each composition, the electron distribution function is obtained from the solution of the appropriate Boltzmann equation and is then used to calculate the various ionization and excitation rates that appear in the governing equations. The results of the theory are used to study the effects of the various operating parameters on the small signal gain coefficient.

Contents

The flow geometry considered in this investigation is that employed in the experiments of Kan and Whitney.[1] For the flow rates under consideration, convective cooling, and axial variations dominate the flow processes; therefore wall effects are neglected and a one-dimensional analysis is used. Molecules in different quantum states are treated as different species and the conservation equations of mass, momentum, and energy are used to predict, for given inlet conditions, tube dimensions and current density (or power), the gas and electron temperatures, the velocity, the pressure, the number densities of the electrons and the various excited states, and the small signal gain coefficient as a function of position along the tube.

The kinetic model employed in the analysis takes into consideration the following processes: 1) CO ionization; 2) electron excitation of the vibrational states of CO and N_2 up to $v = 8$; 3) vibration-translation (VT) deactivation processes; 4) vibration-vibration (VV) single quantum exchange collisions (CO, CO; CO, N_2; N_2, N_2); 4) Spontaneous radiative decay ($\Delta v = 1, 2$); and 5) stimulated emission and absorption processes. The VV rates were based on the formulation of Rockwood et al.[2] The Rockwood expression for the long range part has been modified to give the correct temperature dependence on both the magnitude of the rate and half-width of the Gaussian function. The constants that appear in the Rockwood formulation were adjusted to fit the 100 K experimental data of Wittig and Smith.[3] The VT rates were obtained from SSH theory and available experimental data. The SSH expression was modified by the factor

$$\exp\left[4(DX)/\pi kT + 16D/3\pi^2 kT\right]$$

obtained by Shin[4] for a Morse interaction potential; here D is the well depth of the Morse potential, X is related to the argument of the exponential factor that appears in the SSH theory, k is the Boltzmann constant, and T is the temperature. The small signal gain coefficient is taken from Ref. 5 and includes both pressure and Doppler broadening effects. Forty-five levels of CO and twenty-five levels of N_2 were employed in the calculations.

Because of the symmetry of the flow geometry,[1] the calculations are performed for only half of the tube. The average small signal gain coefficient $\bar{\gamma}_v$ for a particular transition $v \rightarrow v-1$ is

$$\bar{\gamma}_v = \frac{1}{L}\int_0^L \gamma_v \, dx$$

where L is the length of the tube. The base case corresponds to $P_{IN} = 27$ torr, $\dot{m}_{CO} = 0.392$ g/sec, $\dot{m}_{N_2} = 0.168$ g/sec, $\dot{m}_{H_e} = 1.48$ g/sec, $J = 0.1$ amp, $T_{IN} = 77$ K. Because the entrance temperature was not reported, the calculations for the base case were carried out for $T_{IN} = 77$ and 150 K. Figure 1 shows the axial variation of the small signal gain coefficient of three selected

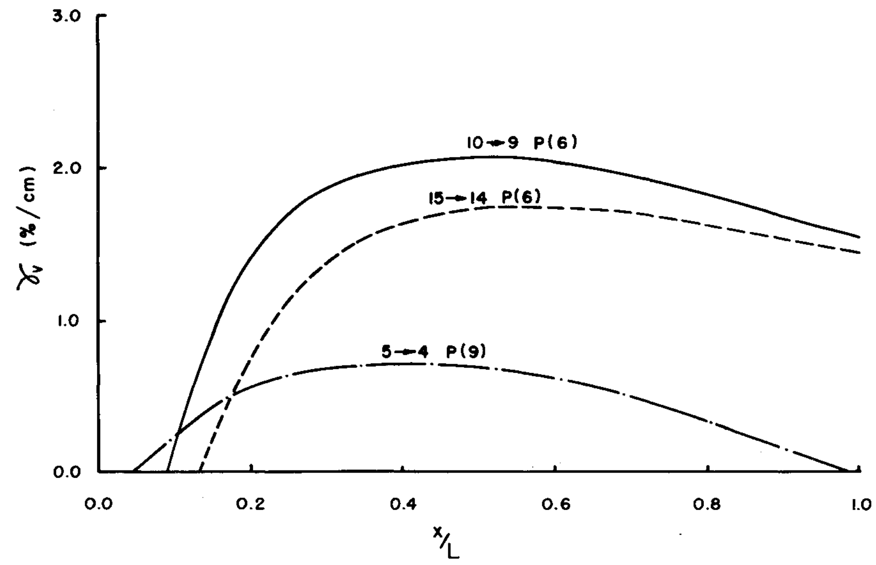

Fig. 1 Variation of the maximum gain for selected transitions with distance into the cavity.

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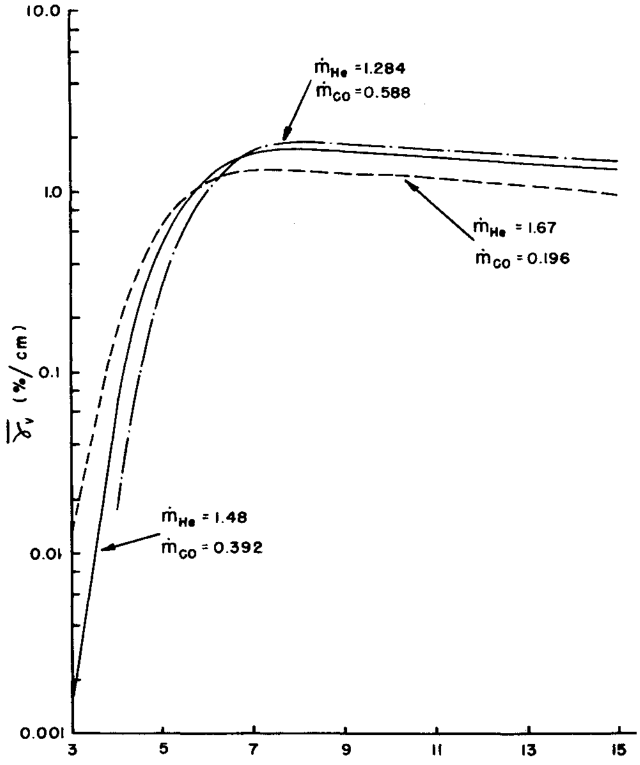

Fig. 2 Influence of CO-He flow rate ratio on the maximum average small signal gain ($\dot{m}_{N_2}$ = 0.168 g/sec).

transitions (the gain coefficient is positive for more than one rotational level J); the values indicated in Fig. 1 are those for the J values which result in maximum $\bar{\gamma}_v$). The $3 \rightarrow 2$ transition (not shown) is the lowest transition to develop a positive average small signal gain. As the VV pumping continues to populate the upper levels, the gains for the $10 \rightarrow 9$ and $15 \rightarrow 14$ transitions become positive and rise rapidly toward their peak values. The larger gains of these lines is due to the almost level distribution produced among the intermediate levels. Figure 2 shows the influence of CO mass flow rate on the maximum average small signal gain for each transition. The CO flow rate is adjusted in such a way that the combined helium and CO flow rates remained constant and all other operating conditions remained the same. One would expect that the average small signal gain coefficient would increase with an increase in

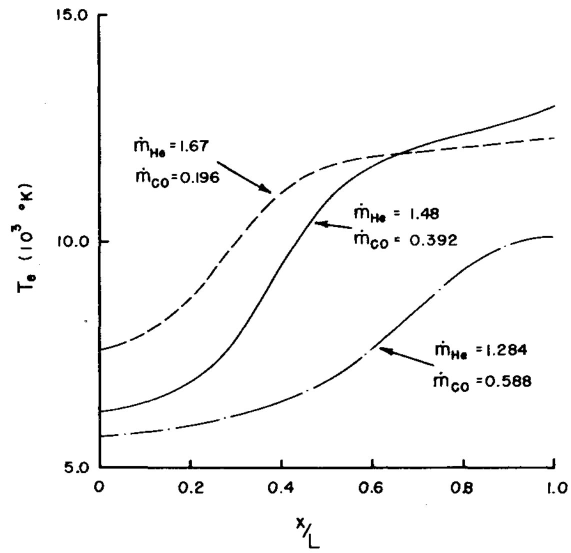

Fig. 3 Influence of CO-He flow rate ratio on the variation of electron temperature with distance into the cavity ($\dot{m}_{N_2}$ = 0.168 g/sec).

Table 1 Comparison of J values of observed transitions and calculated maximum gain coefficients

Vibrational band	Transitions		
	Ref. 1	T_{IN} = 77 K	T_{IN} = 150 K
$5 \rightarrow 4$	P(12)	P(10)	P(14)
$6 \rightarrow 5$	P(12)-P(13)	P(8)	P(12)
$7 \rightarrow 6$	P(11)-P(13)	P(8)	P(12)
$8 \rightarrow 7$	P(12)	P(7)	P(12)
$9 \rightarrow 8$	P(11)-P(12)	P(7)	P(11)
$10 \rightarrow 9$	P(10)-P(12)	P(7)	P(11)
$11 \rightarrow 10$	P(10)-P(11)	P(7)	P(11)
$12 \rightarrow 11$	P(10)	P(7)	P(11)
$13 \rightarrow 12$	P(9)-P(11)	P(7)	P(11)
$14 \rightarrow 13$	P(9)-P(10)	P(7)	P(11)
$15 \rightarrow 14$	P(9)	P(7)	P(11)

the number density of CO. As is seen from Fig. 2, this is not the case for the lower levels. The behavior may be understood by referring to Fig. 3 which shows a plot of the electron temperature vs distance. It is seen that the electron temperature increases with a decrease in CO flow rate with the result that the excitation rates of the lower levels are much higher at the lower CO flow rates. Lower CO flow rates are also characterized by lower gas temperatures. These competing effects combine to give the behavior indicated in Fig. 2. The preceding illustrate the importance of utilizing the correct distribution function that corresponds to the local E/N in calculating the excitation and ionization rates; assuming a constant E/N throughout the tube, (which, evidently, is standard practice in CO modeling) would have resulted in a completely different behavior.

The maximum average small signal gain coefficient for each transition at constant CO flow rates and constant He and N_2 flow rates is essentially independent of the mixture ratio for a range of N_2 flow rates from 0 to 0.336 g/sec. On the other hand, for a constant He flow rate and constant combined CO and N_2 flow rates with N_2 ranging from 0–0.364 g/sec, the maximum average small signal gain coefficients increase slightly with N_2 flow rates for the lower levels $v \leq 6$ and decreases slightly for the higher levels.

Table 1 compares the J values corresponding to the maximum average gain with those of the observed transitions in Ref. 1. Because of the uncertainty in the entrance temperature, calculations for $T_{IN} = 77$ K and 150 K are included. It is seen that an entrance temperature between 77 and 150 K would lead to results which are in good agreement with experiment. These results also indicate that lower temperatures tend to shift the maximum average gain to lower J numbers.

It should be pointed out that the data of Ref. 3 does not agree with the theory of Jeffers and Kelly[5] at the low temperatures considered in this work. Our preliminary calculations for CO-He laser systems suggest that this discrepancy has a very small effect on the maximum average small-signal gain coefficient.

References

[1] Kan, T. and Whitney, W., "Forced Convective Flow Carbon Monoxide Laser," *Applied Physics Letters*, Vol. 21, No. 5, Sept. 1972, pp. 213–215.

[2] Rockwood, S. D., Brau, J. E., Proctor, W. A., and Canavan, G. H., "Time-Dependent Calculations of Carbon Monoxide Laser Kinetics," *IEEE Transactions on Quantum Electronics*, Vol. QE-9, No. 1, Jan. 1973, pp. 120–129.

[3] Wittig, C. and Smith, I. W. M., "Vibrational Relaxation of Carbon Monoxide ($4 \leq v \leq 10$) at $T = 100°$K," *Chemical Physics Letters*, Vol. 16, No. 2, Oct. 1972, pp. 292–295.

[4] Shin, H. K., "Vibrational Relaxation in CO+He at Low Temperatures," *The Journal of Chemical Physics*, Vol. 55, No. 11, Dec. 1971, pp. 5233–5234.

[5] Jeffers, W. Q. and Kelly, J. D., "Calculations of V-V Transfer Probabilities in CO-CO Collisions," *The Journal of Chemical Physics*, Vol. 55, No. 9, Nov. 1971, pp. 4433–4437.